Toy Train

For a set of stations S, define $f_A(S)$ as the set of all stations such that if the train is placed on them, Arezou can play in a way that the train reaches one of the stations in S at some point, regardless of Borzou's moves (therefore $S \subseteq f_A(S)$ by definition). We define $f_B(S)$ similarly.

Initially, let $T = S$, and we iteratively add stations to T such that eventually T becomes equal to $f_A(S)$.

While there exists a station v that satisfies one of the following conditions, we add v to the set T:

1. v is owned by Arezou, and there exists an outgoing track from v that arrives at a station already in T.
2. v is owned by Borzou, and *all* of the outgoing tracks from v arrive at the stations already in T.

Similarly we can compute $f_B(S)$. Notice that both $f_A(S)$ and $f_B(S)$ can be computed iteratively in time $O(n + m)$.

Now let R be the set of all the charging stations. By definition, for every station $v \notin f_A(R)$, Borzou can win the game if the train is initially placed on v.

Therefore, we can solve the problem as follows:

1. If $f_A(R)$ is the set of all stations, Arezou can win the game for all initial stations.
2. Otherwise:
    1. Let X be the set of all stations not in $f_A(R)$.
    2. Borzou can win the game if the initial stations are in $f_B(X)$.
    3. Remove $f_B(X)$ from the graph and solve the problem recursively for the remaining stations.

This algorithm runs in time $O(nm)$.